Engineering Economics

Greater knowledge of engineering economics and cost analysis is expected of industrial engineers than of other engineers. On the 110-question FE Exam, expect 10–15 questions on engineering economics. Specifically, expect questions in discounted cash flows, types and breakdown of costs, cost analyses, accounting, cost estimation, depreciation and taxes, and capital budgeting.

ENGINEERING ECONOMICS

The Engineering Economics section of the NCEES’s *Fundamentals of Engineering (FE) Supplied-Reference Handbook* gives nomenclature and definitions; compound interest formulas and tables; depreciation schedules; and brief discussions of rate of return, breakeven analysis, inflation, and benefit-cost analysis. We will clarify those materials and review essential concepts not covered in the Handbook.

Compound Interest

The symbol $A$, identified in the handbook as the “uniform amount per interest period,” is the familiar quantity better known as the equal-payment-series amount. A loan of $P$ dollars can be repaid in $n$ end-of-period installments of $A$ dollars each. Revenues of $A$ dollars per year for $n$ years have present worth $P$ dollars, or, at time $n$, future worth (also called compound amount) $F$ dollars. It is customary to use the end-of-year convention; cash flows (inflows or receipts positive, outflows or disbursements negative) that flow within a year are considered to flow at the end of the year. Two compound interest formulas, although listed in the handbook, should be memorized:

$$F = P(1 + i)^n \quad \text{or} \quad P = F\beta^n$$

where $\beta = 1/(1 + i)$ and $P = A(1 - \beta^n)/i$

$\beta^n$ is the single-payment present worth factor ($P/F$, $i$, $n$), and $(1 - \beta^n)/i$ is a computation-convenient form of the “pain” factor ($P/A$, $i$, $n$). If you program the “pain” factor into your hand calculator, you probably won’t need to use any compound interest tables during the FE exam.
Nominal and Effective Interest Rates

The handbook gives a formula for the effective (annual compound) interest rate \( i_e \) that corresponds to a nominal interest rate \( r \) when there are \( m \) compounding intervals per year:

\[
\text{[EQ]} i_e = 1 + \frac{r}{m} \]

Remember that \( i = r/m \) is the interest rate per period. For example, \( i = 1.5\% \) interest on the unpaid balance of a credit card account corresponds to \( r = 0.015 \times 12 = 18\% \) as a nominal monthly compound interest rate. The effective interest rate is \( i_e = (1 + 0.015)^{12} - 1 = 19.56\% \).

Compound interest formulas given in the handbook for continuous compounding are merely the usual formulas with \( e^r \) substituted for \((1 + i)\). Here, \( r \) is the limit of the nominal interest rate as the number of compounding intervals goes to infinity. Formulas in this form are handy if you are given the interest rate as a nominal continuous compound interest rate. However, you could also determine \( i_e = e^r - 1 \) and use the regular formulas.

**Example A.1**

If the interest rate on a savings account is 8% compounded annually, and you deposit $100, what will it grow to in half a year?

**Solution**

From \((F/P, r, n) = e^m\), we have \( F = P(F/P, r = 8\%, 0.5) = 100e^{0.04} = $104.08\). But from the usual formula \( F = P(F/P, i_e, 0.5) \) with \( i_e = e^{0.08} - 1 = 0.083287068\), we have \( F = 100 (1 + i_e)^{0.5} = $104.08\). The usual formulas work for noninteger values of \( n \).

**Interest Amount and Amortization**

Under any loan repayment schedule, a portion of each payment is interest. A loan repaid in more than one installment can be viewed as a series of one-period loans. For example, if a $2,332.10 payment is made at the end of the first year on a $10,000 loan at 8% compound interest, then the interest for the first year is \( 0.08 \times 10,000 = $800 \). The $1,532.10 remainder of the payment reduces the principal of the loan, which becomes $8,467.90. Whatever the amount of the payment at the end of the second year, the interest for the second year is \( 0.08 \times 8,467.90 = $677.43 \), and the remainder of the payment again reduces the principal. If every payment is $2,332.10, determined by calculating \( A \) for the loan, then this payment amount is the amount, to the nearest cent, needed to bring the principal down to zero after \( n \) payments.
An amortization schedule is a document that walks this procedure through, recording the balance (principal), the payment amount, the interest on the balance, the balance (principal) reduction, and the new balance for each period.

**Income Tax and Depreciation**

A table of MACRS depreciation schedules is given in the handbook. Its entries are percentages, and they sum to 100%. For instance, a MACRS-3 schedule has factors 33.3, 44.5, 14.8, and 7.4; thus, the depreciation charges for a $100 depreciable investment are $33.30 the first year, $44.50 the second year, and so forth. Depreciation charges are not actual costs. For tax accounting, the actual capital expenditures are ignored, and the corresponding depreciation charges substitute for them. If the income tax rate is $T$, then a depreciation charge $D_j$ in year $j$ reduces the tax obligation by the amount $TD_j$. Thus, for instance, if you paid $5,000 for a machine and it is depreciated on a MACRS-3 schedule, for tax purposes, you paid $0.3330 \times 5000 = 1,665$ the first year and will pay further amounts in subsequent years.

Expenditures that can be expensed (e.g., repairs) effectively have a 100% depreciation charge the first year.

**Worth, Equivalence, and Rate of Return**

Engineering economic decisions are made on the basis of worth unless they involve alternatives that can be freely scaled in size. At a given interest rate, all worth measures are equivalent: if Alternative $A$ has positive (or negative, or zero) present worth, it correspondingly has positive (or negative, or zero) future worth for any $n$ and also positive (or negative, or zero) annual worth for any $n$. If $P$ is the present worth of an alternative, and $F$ is the future worth for time $n$ of the same alternative, then $F = P(1 + i)^n$. If $A$ is the annual equivalent, then $P = A(P/A, i, n)$. Thus, for any problem, the worth measure most convenient to calculate for that problem can be used; other worth measures can be computed using the mentioned formulas, if necessary. If present worth favors Alternative $A$ over Alternative $B$, then future worth and annual worth will also favor Alternative $A$ by the same ratio.

In making engineering economic decisions, each alternative is represented by a set of cash flows. Generally, we prefer the alternative whose set of cash flows has maximum worth. However, if alternatives can be scaled up or down (e.g., comparing two types of paving per mile of road), we may prefer the alternative whose set of cash flows maximizes an efficiency measure such as rate of return. Rate of return is a measure of earning efficiency. It is defined as the interest rate that makes a set of cash flows have zero worth. For example, if an opportunity calls for an investment of $I$ dollars followed by annual returns of $A$ dollars per year for $n$ years, and a salvage value of $S$ dollars at time $n$, then the present worth is $P = -I + A(P/A, i, n) + S \beta^n$, where $\beta = 1/(1+i)$. The value of $i$ for which $P = 0$ is the rate of return; it can be determined by numerical search. Your hand calculator may have a rate-of-return
routine, but on a multiple-choice exam, you can simply choose the interest rate that comes closest to making $P = 0$.

The minimal attractive rate of return, or MARR, is the interest rate that an enterprise uses for worth calculations. It is greater than the interest rate on borrowed money; it is equal to the rate of return that the enterprise earns in the ordinary course of business. A good opportunity has positive worth at MARR; its present worth can be interpreted as the amount of money by which the opportunity is better than making a similar investment in the ordinary course of business.

Three efficiency measures besides rate of return are in common use. The \textit{payback time}, defined as the earliest time the sum of the positive cash flows permanently exceeds the sum of the negative cash flows, is a measure of illiquidity (i.e., how long funds are tied up). For example, if you knew you could purchase a $300,000 painting now and sell it for billions, but only after waiting 25 years, then, even if the billions were enough after inflation to give a huge rate of return, you might pass up the opportunity.

The \textit{net present value index} (NPVI) is a measure of capital-budgeting efficiency—how well an opportunity conserves scarce funds available for investment. (\textit{Value} in the name of this index is a synonym for \textit{worth}.) It is defined as the net present worth of an opportunity divided by its initial investment. For example, when budgets are tight, an investment with a $325,000 net present worth that requires a $500,000 immediate investment would have a 0.65 NPVI. If there is a limit on how much can be invested this year, this opportunity may be deferred unless sufficient investment capital remains after funding all the opportunities having greater NPVI.

The \textit{benefit-cost ratio} (B/C) is a measure of the ratio of the worth of benefits to the worth of costs and is useful when benefits (or costs, or both) are measured in nonmonetary terms (e.g., number of cases cleared or number of deaths avoided). It is of limited value when the distinction between benefits and costs is merely that between positive and negative cash flows. In that case, B/C is simply NPVI + 1.

\textbf{Inflation}

Every cash flow’s amount is estimated either before inflation (e.g., labor cost, energy cost, sales revenues) or after inflation (e.g., lease payments, loan repayments, tax obligations, depreciation charges). If an amount $W_t$ expected to flow at time $t$ is a before-inflation amount (usually called \textit{indexed}—an amount indexed to time zero), it is understood that the corresponding after-inflation amount $A_t$ (usually called \textit{current}—the amount for which “the check will actually be written”) will be $A_t = W_t (1 + f)^t$. Conversely, the before-inflation amount that corresponds to a given after-inflation amount $A_t$ is given by $W_t = A_t / (1 + f)^t$.

Following the handbook’s nonstandard nomenclature, let before-inflation amounts be discounted using interest rate $i$. Instead of converting after-inflation amounts to before-inflation amounts and then discounting them at $i$, we can discount them at an after-inflation interest rate $d$ (usually called a “combined” rate—it combines the effects of compound interest and inflation). The present worth of $W_t$
is \( W_t / (1 + i)^t \). The present worth of the corresponding \( A_t \) is \( A_t / (1 + d)^t = W_t (1 + f)^t / (1 + d)^t \). Since both ways of computing the present worth of the cash flow must give the same answer, it is apparent that \( 1 / (1 + i) = (1 + f) / (1 + d) \), or \( d = (1 + i)(1 + f) - 1 = i + f + if \), as given by the handbook.

**TYPES OF COSTS**

Industrial engineers categorize costs. Costs of a process are often classified into fixed costs \( f \) per time period and variable costs \( v \) per unit of production, so that if the production rate is \( x \) units per period, then the total costs per period are \( f + vx \). Variable costs are expenses that change in proportion to business activity, while fixed costs are not dependent on the level of business activity. Fixed costs are often time dependent, such as rent, salaries, or the cost of starting a production process. The term does not mean the cost will never change over time; it means it won’t change as the volume changes. If you run a printing shop, fixed cost would be rent on the printers and employee salaries. The cost of paper and ink would vary with the volume of print orders and are variable costs.

Another way to categorize costs is direct versus indirect costs. Expenses that are clearly associated with one project or another are called direct costs. Those that are common to the organization and not specifically associated with a particular project are indirect costs.

Table A.1  Direct and Indirect Costs.

<table>
<thead>
<tr>
<th>Direct Costs</th>
<th>Indirect Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salaries and wages of workers on the specific project (i.e., engineers, administrative assistant)</td>
<td>Salaries and wages of general administrative staff (i.e., CEO, accountant, custodian)</td>
</tr>
<tr>
<td>Consultants and subcontractors to accomplish a specific project objective</td>
<td>Professional memberships</td>
</tr>
<tr>
<td>Travel in support of a project</td>
<td>Training or educational expenses</td>
</tr>
<tr>
<td>Materials and supplies used directly by the project (i.e., postage, paper, office supplies)</td>
<td>Equipment used on several projects (i.e., computer printer, forklift)</td>
</tr>
<tr>
<td>Resources used solely for one project (i.e., tanks of oxygen, long distance phone calls)</td>
<td>Utility costs (i.e., water, electric, cell phone)</td>
</tr>
</tbody>
</table>

Often indirect costs are shared among the various projects at an organization. This is also called *overhead* or *burden*. It is the indirect costs associated with a project’s direct cost. This is often done using percentages.

Activity-based costing (ABC) is a cost accounting practice that allocates fixed cost to business activities. It attempts to identify and assign the cost of resources and services according to the actual amount consumed by activity/project. This approach assigns more indirect costs into direct costs compared to conventional costing. It gives the manager a better understanding of a project’s cost.
and productivity. For example, traditional cost accounting might add 10% to the hours billed by each doctor for medical supplies (a fixed overhead rate). On the other hand, ABC would require staff to scan the barcodes of supplies (bandages, medicine, etc.), and the patient is billed for the doctor’s time and actual supplies used.

**INDUSTRIAL COST ANALYSIS**

**Breakeven Analysis**

As the handbook says of breakeven analysis, “By altering the value of any one of the variables in a situation, holding all of the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical.” A common use of breakeven analysis is to compare two alternatives over variable conditions.

**Example A.2**

A hospital is considering leasing a piece of diagnostic equipment at $200,000 per year. The variable cost of the equipment is $100 per patient, and the average insurance plan will pay $200 per patient test on the machine. What is the break-even quantity for this equipment?

**Solution**

We could solve this problem graphically by plotting the line for annual revenues and the line for annual cost on a graph with the number of patients along the X axis and dollars along the Y axis. The point where the two lines cross is the break-even point, where the profit from leasing the machine with that patient load results in zero profit. We can also solve the problem algebraically by taking our cost and dividing by our profit to find our quantity, $Q$.

$$Q = \frac{200,000}{(200 - 100)} = 2000 \text{ patients}$$

Remember, breakeven analysis can be a lower or an upper limit. For example, we must sell at least $X$ units to break even with respect to production costs, and the order must be smaller than $Y$ to prevent excess storage costs. These are examples of a lower limit and an upper limit, respectively. To determine a breakeven point, you set an inequality ($\leq$ or $\geq$) to be an equality (=) and solve.

Recall the present worth formula used as an illustration in the rate-of-return discussion:

$$P = -I + A(P/A, i, n) + S\beta^n, \text{ where } \beta = 1/(1 + i).$$
This is the present worth of an opportunity that has an investment of $I$ dollars, annual returns of $A$ dollars per year for $n$ years, and a salvage value of $S$ dollars at time $n$. Breakeven analysis questions could be asked in which any one of the quantities could be the variable and all the other quantities are fixed. For the opportunity to be worthwhile ($P \geq 0$), we could ask for
- the greatest investment $I$,
- the smallest annual return amount $A$,
- the greatest interest rate $i$,
- the smallest salvage value $S$, or
- the smallest economic life $n$.

Note that if the missing quantity is a cash flow amount such as $I$, $A$, or $S$, solving for it is easy. If the variable is $i$, we have a rate-of-return problem. If the variable is $n$, for this worth formula, it happens that we can solve for $n$. The “pain” factor is $(1 - \beta^n)/i$, so that the worth is linear in $\beta^n$. After solving for that quantity, say $c$, we can compute

$$n = (\log c)/(\log \beta).$$

**Cost Treatment**

To analyze alternative designs or courses of action, the costs and benefits of alternatives must actually be consequences: costs must be unavoidable and benefits otherwise unobtainable. A free good has zero value. For instance, if a project uses scrap that would not otherwise be sold, or utilizes idle labor intervals within a rigid cycle, then the cost of the scrap or the labor is zero.

Except within a rigid cycle, however, labor has a value equal to pay plus benefits. For instance, if a project saves four labor hours per shift, the savings is counted despite not laying off anybody because it is presumed that freed-up labor can be utilized elsewhere.

Sunk costs have zero value. For instance, imagine your company has partially completed a major design project estimated to be worth $500,000 before a major economic downturn. Project costs to date are $300,000, and an additional $60,000 of man-hours will be needed to complete the project. However, with the market economy, the final project can be sold for only $50,000. Considering these figures, the project should not be completed unless the sales price increases about $60,000 or design costs go below $50,000. The $300,000 already spent is a sunk cost, a money that would be available if the alternative had not been selected originally.

The cost of a good or service can be simply the variable cost if there are no significant setup costs. In other situations, a fixed amount can be added to the cost, such as a “handling charge” when you buy concert tickets. It is the same regardless of the number of tickets you buy. A third situation is when an indirect or overhead rate is used. For this, the cost is its variable cost plus an additional percentage. An
example of this is when a real estate agent charges 6% of a house’s selling price as a commission to cover the agent’s expenses.

**ACCOUNTING**

Accounting is the production of financial records about an organization. This is often done via financial statements that show in money terms how well an entity is performing. Common financial statements include a

- balance sheet (statement of financial position), which reports on the entity’s assets, liabilities, and ownership equity at a given point in time;
- profit and loss statement (statement of comprehensive income), which reports on the entity's income, expenses, and profits over a period of time providing information on operations; and
- statement of cash flows, which reports on a company's cash flow activities, particularly its operating, investing, and financing activities.

There are two commonly used accounting methods, the cash basis and the accrual basis, used to track income and expenses. The cash basis of accounting records revenue when cash is received and records expenses when cash is paid. For example, the teenager living next door sends me a text message each time she mows my yard (invoicing me) and records my payment on a spreadsheet when I pay (payment is received). When she buys gasoline or oil for the mower (incurs debt), she updates her spreadsheet. She is using the cash basis. The accrual method records income when it is earned and records deductions when expenses are incurred. For a business invoicing for an item sold or work done, the corresponding amount will appear in the books even though no payment has yet been received. Debts owed by the business show as they are incurred, even though they may not be paid until later.

In double entry bookkeeping, debits and credits are entries made in account ledgers to record changes in value due to business transactions. If the transaction is a debit, an entry is made on the left; if the transaction is a credit, an entry is made on the right side of the account's ledger. Each transaction's debit entries must equal its credit entries. This is captured by the accounting equation

\[
\text{assets} = \text{liabilities} + \text{equity} \quad (A = L + E).
\]

The difference between the total debits and total credits in a single account is the account's balance. For the entity as a whole, the totals of debit balances and credit balances must be equal as shown on the balance sheet.
Suzie started her company by investing $9,000. She currently has $7,000 in inventory and $2,000 in cash. This is shown on the assets side of the balance sheet. The same total is shown on the liabilities side of the balance sheet.

<table>
<thead>
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<th>LIABILITIES</th>
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<td>Cash</td>
<td>$ 2,000</td>
</tr>
<tr>
<td>Inventory</td>
<td>$ 7,000</td>
</tr>
<tr>
<td>$ 9,000</td>
<td></td>
</tr>
<tr>
<td>Surplus</td>
<td>$ 5,000</td>
</tr>
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<td>Owner equity</td>
<td>$ 4,000</td>
</tr>
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For each transaction, you each post a credit and a debt. If Suzie takes a loan of $500, it would be shown as $500 increase in cash and $500 in loans payable. The new balance sheet would be

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</tr>
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**COST ESTIMATION AND BUDGETING**

A cost estimate is the approximation of the cost of a program, project, or operation. The cost estimate is the product of the cost estimating process. The problem of cost overrun can be avoided with a credible, reliable, and accurate cost estimate. A contingency may be included in an estimate to provide for unknown costs, which are indicated as likely to occur by experience but are not identifiable. Factor estimating is one method of estimating; it involves taking the known cost of a similar facility and factoring the cost for size or scope. For example, if the company spent $750,000 for a 10,000 square-foot warehouse, a rough cost estimate for an 8,000 square-foot warehouse would be calculating the scope (8,000/10,000 sq ft = 80%) and multiplying that by the original cost ($750,000 × 0.80 = $600,000). Cost modeling is another common technique. In this technique, the estimator models the various parameters of the facility (number of loading docks, restrooms, offices, storage racks, etc.) and applies costs to each to derive the total budget. Capital budgeting is the planning process to determine an organization's long-term investments in large expenditures such as machinery, plants, production line, or research development projects.
PROBLEMS

A.1. A $105,000 mortgage loan is to be repaid in 360 monthly installments at 9% nominal interest. The amount of each installment is most nearly
   a. $790.
   b. $840.
   c. $845.
   d. $880.

A.2. To help pay for a new pipeline extension, a company borrows $14,250 at 12% annual compound interest to be repaid in two equal annual installments. Interest on the loan is deductible. The amount of interest paid in the second year is most nearly
   a. $870.
   b. $900.
   c. $1850.
   d. $6690.

A.3. Assume you have two alternatives for a worn-out machine. You can replace the machine by paying $5000 for a machine that has a 3-year MACRS depreciation recovery period, or you can repair the existing machine. The maximum amount that you could afford to pay for the repair if 15% interest rate is used in after-tax analysis, and income tax rate is 40%, is most nearly
   a. $1500.
   b. $4800.
   c. $5300.
   d. $5800.

A.4. Annual 4% inflation is expected. A remote control panel can be leased for five years at a fixed cost of $3000 per year, paid at the beginning of each year. It will save 125 labor hours per year. A labor hour is currently valued at $22.50. At 14% interest before inflation, the present worth of this opportunity is most nearly
   a. −$640.
   b. −$580.
   c. −$180.
   d. $390.
A.5. A yogurt-making process ends with filling and packaging. You can purchase a new filler machine that can fill up to 700 liters per hour. Under this option (option A), fixed costs for filling and packaging will be $89 per hour, and variable costs will be $0.067 per liter. Alternatively, you can purchase a combination machine that integrates filling with packaging and can handle up to 500 liters per hour. Under this option (option B), fixed costs will be $105 per hour and variable costs will be $0.024 per liter. A third alternative (option C) is to contract out the filling and packaging at $0.50 per liter. Production will not exceed demand. What are the breakeven points in demand (liters per hour)? Which of the various options are best?

a. Choose C below 200, B between 400 and 500, otherwise A.
b. Choose B below 400, A above 400.
c. Chooses C below 200, A above 200.
d. Choose A below 400 and above 500, otherwise B.

A.6. Zach operates a trucking company. As the price of gasoline increases, you would expect all of the following costs to increase EXCEPT

a. direct costs.
b. total costs.
c. shipment costs using activity-based costing.
d. indirect costs.

A.7. Sales and the various expenses that were incurred during the year would appear on which of the following financial statements?

a. Balance sheet
b. Profit and loss statement
c. Indirect cost report
d. Debit and credit report

A.8. A company purchases a small motor from a vendor for $7.50/motor. They are considering manufacturing the motors in-house. The company has excess capacity that could be used for the motor production. The manufacturing would have a direct material and labor cost of $2.85, a fixed one-time setup cost of $500, and a variable overhead cost of $0.50. At what volume are you most nearly indifferent between making or buying the motors?

a. It is cheaper to make motors regardless of cost.
b. 120 motors
c. 150 motors
d. 1000 motors
A.9. To celebrate passing the FE exam, Richard buys 100 shares of a stock called U-Neva at $60 per share. He will sell the stock when its market price has doubled. He expects the stock price to increase at a steady rate yearly. How much must the stock grow for Richard to be able to sell the stock and double his money in six years?
   a. 6%
   b. 10%
   c. 12%
   d. 15%

A.10. Katie makes quarterly deposits in a savings account that earns 9% interest compounded monthly. Her effective interest rate per quarter is MOST closely
   a. 0.75%.
   b. 2.27%.
   c. 3.0%.
   d. 9.0%.
A.1 c. \( r = 0.09, m = 12, \) so \( i = 0.09/12 = 0.0075. \) \( (P/A, i, 360) = 124.2818656. \)

Given \( P = 105,000, \) the result is \( A = P/(P/A, i, 360) = \$844.85, \) which is near \$845.

A.2 b. The amount of the installments is \( A = 14,250/(P/A, 12\%, 2) = 14,250/ 1.6901 = \$8431.45. \)

The interest for the first year is \( 0.12 \times 14,250 = \$1710.00, \) so the balance is reduced by the remainder \( 8431.45 – 1710 = \$6721.45. \) The new balance is \( 14,250 – 6721.45 = \$7528.55. \) The interest for the second year is \( 0.12 \times 7528.55 = \$903.426. \)

A.3 c. If repair is not chosen, the depreciation changes in years \( \{1, 2, 3, 4\} \) will be \( \$5000 \times \{0.333, 0.445, 0.148, 0.074\}, \) and the corresponding tax relief amounts will be \( T \) times these amounts, where \( T = 0.40. \) The resulting tax relief amounts are \( \{\$666, \$890, \$296, \$148\}. \) With \( \beta = 1/1.15, \) the present worth of the tax relief will be \( 666\beta + 890\beta^2 + 296\beta^3 + 148\beta^4 = \$1531.34. \)

On the other hand, if repair at cost \( X \) was chosen, the cost would be expensed. The tax relief in the first year would be \( 0.40X \) dollars, and the present worth of the tax relief would be \( 0.40X\beta = 0.347826087X \) dollars. You can afford to pay \( X \) dollars for the repair if the present worth of the net after-tax cost (actual cost less tax relief) for repair does not exceed that for replacement: \( X – 0.347826087X \leq \$5000 – 1531.34 \Rightarrow X \leq \$5318.61. \)

**Note:** In this problem, the tax rate actually is irrelevant, as can be seen by carrying \( T \) as a symbol and letting it cancel out of the inequality.

A.4 d. Given \( i = 14\% \) and \( f = 4\%, \) we have \( d = 0.14 + 0.04 + 0.14 \times 0.04 = 18.56\% \) for discounting after-inflation cash flows. The annual labor savings amount is \( 125 \times 22.50 = \$2812.50 \) before inflation. The present worth of the opportunity is \( P = –3000(P/A, d, 5) + 2812.50(P/A, i, 5) = -3000(3.087913881) + 2812.50(3.433080968) = \$391.80. \)

**Note:** Remember labor costs are estimated without inflation.

A.5 a. Let \( x \) be the production rate. In dollars per hour, the total of fixed and variable costs for the various options are \( C^{(A)} = 89 + 0.067x, \) \( C^{(B)} = 105 + 0.024x, \) and \( C^{(C)} = 0.50x. \)

Obviously \( C \) is cheapest for small \( x. \) For \( C \) and \( A \) to have the same total costs, we have \( 0.50x = 89 + 0.067x \Rightarrow x = 205.54, \) and for \( C \) and \( B \) to have the same total costs, we have \( 0.50x = 105 + 0.024x \Rightarrow x = 220.59. \) In the neighborhood of these \( x \) values, \( A \) is cheaper than \( B. \) For \( A \) and \( B \) to have the same total costs, we have \( 89 + 0.067x = 105 + 0.024x \Rightarrow x = 372.09. \) For greater \( x, \) \( B \) is cheaper, but \( B \) can handle only up to \( x = 500. \) Therefore, \( A \) is the only option above 500.

**Note:** If the demand exceeded 700, you would produce 700 under option \( A, \) and the excess demand would presumably be filled under option \( C. \) This is irrelevant to choosing a response because none of the listed responses mention \( C \) for high demand. Sketching graphs of total cost versus demand would probably be the easiest approach to solving this problem.
A.6  **d.** The cost per mile will go up with a rise in gasoline prices. The amount of gas used is a direct cost; it will increase.

Total costs = direct cost + indirect cost

If the direct cost increases, the total cost will also. Shipment costs using activity-based costing will increase directly as the gasoline is charged to the various shipments. For a different type of business, it might be difficult to determine the direct cost of gasoline, in which case it would be included in indirect costs. For a business such as shipping, we can assign the costs and would not put them into indirect cost.

A.7  **b.** Balance sheet and a profit and loss statement are actual financial statements. The other two responses are not and can be quickly eliminated from consideration. A balance sheet shows where the company currently stands, while the profit and loss statement describes recent production operations including sales and expenses.

A.8  **b.** The point of indifference is where the two options are equal. Buying $x$ motors would cost $7.50x$. For making motors, you take the fixed costs ($500) and add the variable costs ($2.85 + 0.50)x$. So $7.50x = 500 + (3.35)x$ at the point of indifference. Solving for $x$, you get 120 motors.

A.9  **c.** We know $P =$ $6,000 and $F =$ $12,000 (double our current value), $N =$ 6 years, and $i$ is unknown. Our formula is $F = P(F/P, i, N)$, so we need a value from the table of 2. Looking up the values of the responses provided, we find 6% is 1.4185, 10% is 1.7716, and 12% is 1.9738: thus, 12% is the closest value. The problem can also be solved using $F = P(1 + i)^N$ and solve for $i$.

A.10  **b.** It is important in problems like this to watch the time units. We are given $r =$ 9%, $C =$ 3 interest periods/quarter, $K =$ 4 quarters/year, and $M =$ 12 interest periods per year. We need to find $i$ using $i = (1 + 0.09/12)^3 − 1 = 2.27%$. 